BioQuest2012 Project: GapMinder Data Fitting

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Malthus Model

Thomas Malthus

An Essay on the Principle of Population

An Essay on the Principle of Population, as it Affects the Future Improvement of Society with Remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers.


"I SAID that population, when unchecked, increased in a geometrical ratio, and subsistence for man in an arithmetical ratio."

---- Thomas Malthus

Malthus Model:

Assumption: the reproduction rate is proportional to the size of the population

\[ \frac{dP}{dt} = kP, \quad k = \text{growth rate per capita} \]

Solution: \[ P(t) = P(0)e^{kt} \]

\( k > 0 \): exponential growth, \( k < 0 \): exponential decay
The reproduction rate is proportional to the population

\[ P(t + \Delta t) = P(t) + kP(t)\Delta t \]

Solve it we have

\[ P(t) = P_0e^{k(t-t_0)} \]
GapMinder Data: US Population and Exponential Data Fitting

US population: 1820-2010 Raw data

Exponential Data Fitting for years 1820-1870

When Exponential Data Fitting was extended to year 1970:
Logistic Population Model

• Developed by Belgian mathematician Pierre Verhulst (1838) in 1838
• The rate of population increase may be limited, i.e., it may depend on population density

\[ P(t + \Delta t) = P(t) + k(P(t))\Delta t \]

where

\[ k(P(t)) = k_0 \left( 1 - \frac{P(t)}{P_m} \right) P(t) \]

The solution is

\[ P(t) = P_0 e^{k_0(t-t_0)} \frac{P_m}{P_0 e^{k_0(t-t_0)} + (P_m - P_0)} = \frac{P_m}{1 + \left( \frac{P_m}{P_0} - 1 \right) e^{-k_0(t-t_0)}} \]
Logistic Population Model

The solution of the Logistic model

\[ P(t) = P_0 e^{k_0(t-t_0)} \frac{P_m}{P_0 e^{k_0(t-t_0)} + (P_m - P_0)} = \frac{P_m}{1 + (\frac{P_m}{P_0} - 1)e^{-k_0(t-t_0)}} \]

With a data fitting
The solution of the Logistic model

\[ P(t) = P_0 e^{k_0(t-t_0)} \frac{P_m}{P_0 e^{k_0(t-t_0)} + (P_m - P_0)} = \frac{P_m}{1 + \left(\frac{P_m}{P_0} - 1\right) e^{-k_0(t-t_0)}} \]

US 1820-2010 Data Fitting
Potential Homework

- Exponential Data Fitting for US Population Year 1970-2010 and predict 2012’s population, and compare the rates with Sweden’s.
- A pure numerical data fitting

\[ P(t) = P_0 e^{k_2 t^2 + k_1 t + k_0} \]